$m=(c-b)^2y$. $\therefore$ $y=\dfrac{m}{(c-b)^2}$. The value of the work which could be done at the age of $c-x$ years is, $x^2y=\dfrac{mx^2}{(c-b)^2}$.

$\therefore$ the value of total amount of work done between the ages of b and c years is, $\dfrac{m}{(c-b)^2}\displaystyle\int_0^{c-b}x^2dx=\frac{1}{3}m(c-b)$.

Adding this to value given in (1), we have, $\frac{1}{3}m(c-b)+m(b-a)=d$, since d=value of horse's work from a to c years of age.

$\therefore$ $m=\dfrac{3d}{c-3a+2b}$. For any age, $b-n$, between the ages of a and b years, we have, the value of horse, V=value of work done after that age $=mn+\frac{1}{3}m(c-b)=\dfrac{3d}{c-3a+2b}[n+\frac{1}{3}(c-b)]\ldots.(A)$.

For any age, $c-n'$, between the ages of b and c years, we have, value of horse,

$$V'=\frac{m}{(c-b)^2}\int_0^{n'}x^2dx=\frac{mn'^3}{3(c-b)^2}=\frac{dn'^3}{(c-b)^2(c-3a+2d)}\ldots.(B).$$

Ex. Let us suppose that a horse is worth \$100 at five years of age, and that he begins to weaken at 18 years of age, becoming worthless at 20. Then, $a=5$, $b=18$, $c=20$, and $d=100$. Substituting (A) becomes,

$V=\dfrac{100(3n+2)}{41}$, (A), and (B), becomes, $V'=\dfrac{25n'^3}{41}\ldots.(B)$. By substituting for n and n' their successive integral values, we construct the following table:

years	value	years	value	years	value
5	\$100	10	\$63.41	15	\$26.82
6	92.68	11	56.09	16	19 51
7	85.36	12	48.78	17	12.19
8	78.04	13	41.46	18	4.87
9	70.73	14	34.14	19	60
				20	00

Also solved by *F. P. MATZ.* and *G. B. M. ZERR.*

29. Sugested by MANSFIELD MERRIMAN, C. E., Ph. D., Professor of Civil Engineering, Lehigh University, South Betholehem, Pennsylvania.

Solve neatly the equations: $\dfrac{y(1+x^2)}{x(1+y^2)}=a\ldots.(1)$, and $\dfrac{y^4(1+x^8)}{x^4(1+y^8)}=b\ldots.(2)$.

I. Solution by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.

Write (1) and (2), respectively,

$$x+\frac{1}{x}=a\left(y+\frac{1}{y}\right)\ldots.(3),\text{ and }x^4+\frac{1}{x^4}=b\left(y^4+\frac{1}{y^4}\right)\ldots.(4).$$

The second power, and the fourth power, of (3) give respectively,

$x^2+\frac{1}{x^2}=a^2\left(y+\frac{1}{y}\right)^2-2\ldots(5)$, and $x^4+\frac{1}{x^4}+4\left(x^2+\frac{1}{x^2}\right)+6$

$=a^4\left(y+\frac{1}{y}\right)^4\ldots(6)$. From (6), by means of (5), we have $x^4+\frac{1}{x^4}+4$

$\left[a^2\left(y+\frac{1}{y}\right)^2-2\right]+6=a^4\left(y+\frac{1}{y}\right)^4\ldots(7)$.

From (4) and (7), $\left(y+\frac{1}{y}\right)^4-\left(\frac{4a^2}{a^4-b}\right)\left(y+\frac{1}{y}\right)^2=-\frac{2}{a^4-b}\ldots(8)$.

$$\therefore\quad y+\frac{1}{y}=\frac{2a^2\pm\sqrt{[2(a^4-b)]}}{a^4-b}=\sqrt{u}\ldots(9).$$

From (9), $y=\frac{1}{2}[\sqrt{u}\pm\sqrt{(u-4)}]\ldots(10)$. From (3), by means of (9), we deduce $x=\frac{1}{2}[a\sqrt{u}\pm\sqrt{(a^2u-4)}]\ldots(11)$.

II. **Solution by G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia, and P. S. BERG, Apple Creek, Ohio.**

We may write the expressions as follows:

$\frac{1+x^2}{x}=a\left(\frac{1+y^2}{y}\right)$, and $\frac{1+x^8}{x^4}=b\left(\frac{1+y^8}{y^4}\right)$, or $x+\frac{1}{x}=a\left(y+\frac{1}{y}\right)$, and $x^4+\frac{1}{x^4}$

$=b\left(y^4+\frac{1}{y^4}\right)$. Let $x+\frac{1}{x}=z$, and $y+\frac{1}{y}=v$.

$\therefore\quad z=av\ldots(1)$, $z^4-4z^2+2=b(v^4-4v^2+2)\ldots(2)$.

Substitute (1) in (2), we get, $a^4v^4-4a^2v^2+2=bv^4-4bv^2+2b$.

$\therefore\quad (a^4-b)v^4-4(a^2-b)v^2=2(b-1)$.

$$\therefore\quad v^2=\frac{2(a^2-b)}{a^4-b}\pm\frac{\sqrt{[2(a^2-b)^2+2b(a^4+1)]}}{a^4-b}=u \text{ suppose.}$$

$\therefore\quad v=\pm\sqrt{u},\ z=\pm a\sqrt{u}$.

$\therefore\ x+\frac{1}{x}=\pm a\sqrt{u}$, or $x^2\pm a\sqrt{u}x=-1$, and $y+\frac{1}{y}=\pm\sqrt{u}$, or $y^2\pm\sqrt{u}y=-1$

$\therefore\quad x=\pm\frac{1}{2}(a\sqrt{u}+\sqrt{[a^2u-4]})$, $y=\pm\frac{1}{2}(\sqrt{u}+\sqrt{[u-4]})$, which are the correct results, and the same as given by the proposed in a previous number of the MONTHLY.

NOTE—A neat solution to No. 26, Algebra, was received from Jno. B. Fraught, A. B., Instructor in Mathematics, Indiana University, after copy had been sent to the Publishers.

PROBLEMS.

29. **Proposed by ARTEMAS MARTIN, LL. D., U. S. Coast and Geoditic Survey Office, Washington, D. C.**